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ANALYTIC REPRESENTATION OF THE JACCHIA 1977 MODEL ATMOSPHERE
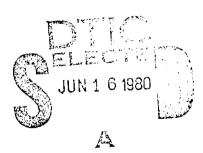
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Modified Jacchia-Walker temperature profiles are developed to accurately represent the Jacchia 1977 temperature profiles above 125 km. The diffusive equilibrium equations are analytically integrable for these profiles, greatly reducing the need for tabular storage of solutions.

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PREFACE

The author wishes to acknowledge the assistance of the following individuals:

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Analytic Representation of the Jacchia 1977 Model Atmosphere.

1.   INTRODUCTION

The diffusion equations for the Jacchia 1977 (J77) density model[1]
cannot be integrated in closed form. Hence numerical results must be
tabulated, requiring extensive storage, if all molecular constituents are
tabulated individually. Fortunately this storage requirement can be
reduced considerably if one notes that the solutions for constituents in
diffusive equilibrium can be expressed as[2]:

$$n_i (h, T_\infty) = n_i (h_0, T_\infty) \left[\frac{T(h_0, T_\infty)}{T(h, T_\infty)} \right]^{1+\alpha_i}$$

$$\cdot \exp \left[-M_i F (h_0, h, T_\infty) \right]$$

where

n_i = density of ith molecular constituent

h = height

T_∞ = exospheric temperature

T = local temperature

α_i = i^{th} constituent thermal diffision coefficient

M_i = i^{th} constituent molecular mass

$F = \int_{h_0}^{h} g(z)/[R^*T(z,T_\infty)] \, dz$

$g(z)$ = acceleration of gravity at height z.

$R^* = 8.31432 \times 10^3 \ J(Kg\text{-}mol)^{-1}/K$

In general one then need tabulate only F as a function of h and T_∞ and
the n_i at the boundary h_0. If h_0 is chosen to be the homopause,
100km, only one n_i need be stored, from which the others are derived.

In this paper we explore alternative profiles for a large portion of the
diffusive region ($h \geq 125$km) for which F can be expressed analytically,
thus further reducing storage requirements.

2.   TEMPERATURE PROFILES (h $\geq$ 125km)

Since a fully analytic solution for the temperature profiles specified in J77 is not possible it seems worthwhile to explore other forms. In particular the Jacchia-Walker (JW) profile[3]:

$$T_{JW}(h,T_\infty) = T_\infty - (T_\infty - T_0) \, e^{-\sigma\xi}$$

leads to the solution

$$F(h_0, h, T_\infty) = \frac{g_e R_e^2}{R^* (R_e + h_0)^2 \, T_\infty} \left\{ \ln\left[\frac{T(h, T_\infty)}{T_0}\right] + \sigma\xi \right\}$$

where

$$\xi = (h - h_0)(R_e + h_0)/(R_e + h) \quad \text{(geopotential altitude)}$$

$R_e = 6356.766$km

g_e = gravitational acceleration at Earth's surface
    = 9.80665 m/sec^2

$T_0 = T(h_0, T_\infty)$

σ = gradient parameter

For application to J77, it is prudent to choose h_0 = 125km, since this is the inflection point, although the various constituent densities at this height cannot be conveniently derived from one of them, as at 100 km. The gradient parameter may be chosen to fit the exact J77 profile in some fashion, such as to match the slope at 125km; i.e.

$$\sigma = G_0/(T_\infty - T_0) = (dT/dh)/(T_x - T_0)$$

Results from this model deviate somewhat from the exact J77 model. In particular, for an exospheric temperature of 900K the local temperature deviation is 20K at 160km and the mass and argon densities differ by 6% and 12% at 225km.

To obtain better fits the form

$$T = [T_{JW} + f]^{-1}$$

has been explored, where the function f is chosen to retain the closed-form integrability of the diffusion equation. In addition it is desirable that the resulting total temperature fit smoothly the exact J77 profile at $h = h_0 = 125$ km and maintain the inflection point character $(d^2 T/dh^2 = 0)$. It is evidently the lack of this latter characteristic which makes the simple JW form undesirable and possibly accounts for much of the error resulting from its use. These modifications lead to the following boundary conditions at $\xi = 0$.

$$f = df/d\xi = 0$$
$$d^2 f/d\xi^2 = -G_0^2 \zeta/T_0^2$$

where

$$G_0 = dT/dh]_{h = h_0}$$
$$\zeta = \sigma + 2/(R_e + h_0) = G_0/(T_\infty - T_0) + 2/(R_e + h_0)$$

For convenience the functional form is chosen to have an exponentially decreasing character with parameters hopefully related to σ. One such form is

$$f = -\left[G_0 \zeta/(2\beta_1^2 T_0^2) \right] f_{\beta_1}(\xi)$$

where

$$f_\beta(\xi) = e^{-\beta\xi} \left(1 - e^{-\beta\xi} \right)^2$$

and β is an adjustable parameter. To provide further flexibility the form

$$f(\xi) = -\left[G_0 \zeta/(2\beta_1^2 T_0^2) \right] f_{\beta_1}(\xi) + C_2 \xi f_{\beta_2}(\xi)$$

has been chosen. The additional term $j = C_2 \xi f_{\beta_2}$ satisfies

$$j = j' = j'' = 0$$

at $\xi = 0$.

For this profile the solution is given by

$$F(h_0, h, T_\infty) = F_{JW} + K\{C_1 (\sum_{i=1}^{3} a_i q_1^i + 1/3)/\beta_1$$

$$+ C_2 \left[\sum_{i=1}^{3} (\beta_2 \xi a_i - b_i) q_2^i + 11/18\right]/\beta_2^2\}$$

where F_{JW} = Jacchia-Walker solution

$c_1 = -G_0 \, \xi/(2\beta_1^2 T_0^2)$

$a_1 = -a_2 = -1, \ a_3 = -1/3$

$q_i = e^{-\beta_i \xi}$

$b_i = a_i/i$

$K = g_e R_e^2/[R^*(R_e + h_0)^2]$

3. RESULTS

The Fletcher-Powell non-linear function minimization method[4,5] has been used to determine the parameters B_1, B_2, c_2 to minimize the sum of squared residuals

$$Q = \Sigma(T - T_{J77})^2$$

for T_∞ = 500, 700, 900, 1100, 1300, 1500, 1700, 1900. By inspection the results are found to be reasonably well represented by:

$$T_\infty \leq 1100K$$

$$B_1 = 2\sigma\left[1 + 10^{-4}\ (T_\infty - 800)\right]$$

$$B_2 = 0.0215 - 0.005\ (T_\infty - 500)/200$$

$$C_2 = 10^{-5}(0.0566 - 0.08X + 0.04X^2); \quad X = (T_\infty - 900)/200$$

$$1100K \leq T_\infty \leq 1500K$$

$$B_1 = 0.0385 - 0.012y + 0.0123y^2; \quad y = (T_\infty - 1100)/400$$

$$B_2 = 0.0065 + 0.0167y$$

$$C_2 = 10^{-5}(0.0166 - 0.4548y^2)$$

$$T_\infty \geq 1500K$$

$$B_1 = 0.0388 - 0.0045z; \quad z = (T_\infty - 1500)/400$$

$$B_2 = 0.0232 - 0.0040z$$

$$C_2 = -10^{-5}\ (0.4382 + 0.0387z)$$

Mass and constituent densities were computed using the Jacchia-Bass (JB) formulation presented here for the T_∞ region 500K -1900K at 100K steps and the h region 130km - 1000km at 10km steps. The results were compared with those from the exact J77 temperature profiles, except that vertical flux is ignored for H. Table I summarizes maximum % deviations for a 6-constituent gas (O_2, O, N_2, He, Ar, H) with the total mass density given by

$$\rho = \sum_{i=1}^{6} M_i \, n_i /A; \quad A = \text{Avogadro's Number}$$

It should be noted that a true J77 calculation would call for different exospheric temperatures for the different constituents to model the different diurnal phases; here all the n_i are computed for the same T_∞. Argon, with the largest mass, yields the largest disagreement among the constituents. The agreement is quite good, well within typical model-experiment differences such as those published by Forbes, Marcos and Gillette[6], and Sharp and Brag.[7] It should be noted that the maximum deviations are all within the 1100K - 1500K region, which proved to be the most difficult to fit. Outside this region all JB mass densities agree with J77 within 1% at all heights. Maximum temperature deviations are 13K at T_∞ = 1300K, h = 350km. Ouside the 1100K - 1500K region the maximum disagreement is 5K.

A FORTRAN deck encompassing the formulation presented here is available on request.

Table I. Maximum Absolute Value of JB-J77 Deviations (%)

	TOTAL DENSITY	Ar DENSITY
h ≤ 300Km	1.1 (h = 200, T_∞ = 1200)	1.5 (h = 190, T_∞ = 1200)
h ≤ 500Km	2.1 (h = 500, T_∞ = 1300)	5.6 (h = 500, T_∞ = 1300)
h ≤ 1000Km	3.0 (h = 750, T_∞ = 1300)	8.5 (h = 1000, T_∞ = 1300)

4.   CONCLUSIONS

It has been shown that analytically solvable temperature profiles can be adjusted to realistically represent temperature vs. height variations above 125km, allowing one to reduce tabular storage requirements and/or avoid the inconvenience of interpolation. It is therefore recommended that similar profiles be used in future atmospheric density modelling. The possibility of developing analytically integrable profiles below 125km will be studied at a later date.

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